Mixed tensor susceptibility of the QCD vacuum from effective quark-quark interactions

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Abstract

We calculate the mixed tensor susceptibility of QCD vacuum in the framework of the global color symmetry model. In our calculation, the functional integration over gluon fields can be performed and the gluonic vacuum observable can be expressed in terms of the quark operators and the gluon propagator. The mixed tensor susceptibility was obtained with the subtraction of the perturbative contribution which is evaluated by the Wigner solution of the quark gap equation. Using several different effective quark-quark interaction models, we find the values of the mixed tensor susceptibility are very small.

Keywords: QCD vacuum ; Induced vacuum condensate; Mixed tensor susceptibility ;Global color symmetry model; Dyson-Schwinger equations

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1. Introduction

In SVZ sum rules, in order to determine the static properties of hadrons it was suggested to consider two-point correlator functions of quark currents in the presence of an external constant classical field, where nonperturbative effects are taken into account in the so-called vacuum susceptibilities[1]. These induced condensates play important roles in determination of the hadron properties such as the nucleon magnetic moments[1], the isovector axial coupling constant[2, 3, 4], the isoscalar axial coupling constant[4], the pion-nucleon coupling constant[5] and the nucleon tensor charge[6, 7] within this version of SVZ sum rules. In the literature, there are always two kinds of vacuum susceptibility that appear in the conventional two-point treatment of an external current field : one is the induced quark condensate and the other is the induced mixed quark-gluon condensate. For convenience, we refer to the former as the quark condensate susceptibility and the later as mixed condensate susceptibility in this letter.

The vacuum tensor susceptibilities are relevant for the determination of nucleon tensor charge[6, 7]. The value of nucleon charge is related to the first moment of the transversity distribution $h_1(x)$, where $h_1(x)$ is an additional twist-two chirality violating structure function which can be measured in the Drell-Yan process with both beam and target transversely polarized. The previous evaluation of the quark condensate tensor susceptibility were performed in the framework of QCD sum rules[6, 7, 8, 9], the chiral constituent model[10] and global color symmetry model(GCM)[11] respectively. Actually, there still exist uncertainty about this induced susceptibility since different theoretical treatments can give very different results, which should be checked by the future measurement of the transversity distribution $h_1(x)$. Another tensor susceptibility, the mixed condensate tensor susceptibility was only evaluated roughly in Ref[6]. within the two-point function of QCD sum rules. In this letter, we will give the calculation of the vacuum mixed tensor susceptibility within the framework of GCM in the mean field level.

As a truncated DSE-model, GCM is a quite successful four-fermion interaction field theory which can be directly derived through a truncation of QCD[13, 14]. This truncated DSE model has been applied extensively at zero temperature and chemical potential to the phenomenology of QCD[15, 16], including the studies of observables from strong interaction to weak interaction area. Furthermore, the truncated DSE models also have made important progress in the studies of strong QCD at finite temperature and chemical potential[17]. Due to the fact that the evaluation of mixed quark-gluon condensate can be performed in the framework of GCM[12], a method to evaluate the mixed vacuum tensor susceptibility is proposed within this DSE formalism and the numerical results for the mixed tensor susceptibility are given.

2. Formalism

In the chiral limit, the QCD generating functional for quark field in the Euclidean space is

given by

$$\mathcal{Z}[\overline{\eta},\eta]=\int\mathcal{D}q\mathcal{D}\overline{q}\mathcal{D}\overline{w}\mathcal{D}w\mathcal{D}A\exp\left\{-S-S_{gf}-S_g+\int d^4x(\overline{\eta}q+\overline{q}\eta)\right\}, \qquad (1)$$

where

$$S=\int d^4x\left\{\overline{q}\left[\gamma_\mu(\partial_\mu-ig\frac{\lambda^a}{2}A^a_\mu)\right]q+\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}\right\}, \qquad (2)$$

and S_{gf}, S_g are the gauge-fixing and ghost actions respectively. Through introducing a bilocal field $B^\theta(x,y)$ as in [8, 9, 10], the generating functional can be given as

$$\begin{aligned}\mathcal{Z}[\overline{\eta},\eta] &= \exp\left[W_1(ig\frac{\delta}{\delta\eta(x)}\frac{\lambda^a}{2}\gamma_\mu\frac{\delta}{\delta\overline{\eta}(x)})\right]\int\mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}B^\theta(x,y)\\ &\quad\times\exp\left\{-S[\overline{q},q,B^\theta(x,y)]+\int d^4x(\overline{\eta}q+\overline{q}\eta)\right\}, \qquad (3)\end{aligned}$$

where

$$W_1[J^a_\mu]=\sum_{n=3}^{\infty}\frac{1}{n!}\int d^4x_1\cdots d^4x_n D^{a_1\cdots a_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n)\prod_{i=1}^{n}J^{a_i}_{\mu_i}(x_i), \qquad (4)$$

and

$$\begin{aligned}S[\overline{q},q,B^\theta(x,y)] &= \int\int d^4xd^4y\left[\overline{q}(x)G^{-1}(x,y;[B^\theta])q(y)\right.\\ &\quad\left.+\frac{B^\theta(x,y)B^\theta(y,x)}{2g^2D(x-y)}\right], \qquad (5)\end{aligned}$$

with

$$G^{-1}(x,y;[B^\theta])=\gamma\cdot\partial\delta(x-y)+\Lambda^\theta B^\theta(x,y). \qquad (6)$$

The quantity Λ^θ arises from Fierz reordering transformation and is the direct product of Dirac, flavor SU(3) and color matrices

$$\Lambda^\theta=\frac{1}{2}(1_D,i\gamma_5,\frac{i}{\sqrt{2}}\gamma_\nu,\frac{i}{\sqrt{2}}\gamma_\nu\gamma_5)\otimes(\frac{1}{\sqrt{3}}1_F,\frac{1}{\sqrt{2}}\lambda^a_F)\otimes(\frac{4}{3}1_C,\frac{i}{\sqrt{3}}\lambda^a_C). \qquad (7)$$

And $g^2D(x-y)$ is the connected gluon two-point function without quark-loop contributions.

Neglecting $W_1[J^a_\mu]$, we can obtain the GCM generating functional as

$$\mathcal{Z}_{GCM}[\overline{\eta},\eta]=\int\mathcal{D}\overline{q}\mathcal{D}qDB^\theta(x,y)\exp\left\{-S[\overline{q},q,B^\theta(x,y)]+\int d^4x(\overline{\eta}q+\overline{q}\eta)\right\}. \qquad (8)$$

Performing the functional integration over $\mathcal{D}\overline{q}$ and $\mathcal{D}q$ in above equation, we obtain

$$\mathcal{Z}_{GCM}[\overline{\eta},\eta]=\int\mathcal{D}B^\theta(x,y)\exp(-S[\overline{\eta},\eta,B^\theta(x,y)]), \qquad (9)$$

where

$$S[\overline{\eta},\eta,B^\theta(x,y)] = -Tr\ln\Big[\partial\cdot\gamma\delta(x-y)+\Lambda^\theta B^\theta(x,y)\Big] + \int\int\Big[\frac{B^\theta(x,y)B^\theta(y,x)}{2g^2D(x-y)}+\overline{\eta}(x)G(x,y;B^\theta)\eta(y)\Big]. \quad (10)$$

The saddle point of this action is defined as $\delta S[\overline{\eta},\eta,B^\theta(x,y)]/\delta B^\theta(x,y)|_{\overline{\eta}=\eta=0}=0$ and is given by

$$B_0^\theta(x-y)=g^2D(x-y)tr_{\gamma C}[\Lambda^\theta G_0(x-y)]. \quad (11)$$

where G_0 stands for $G[B_0^\theta]$ and the trace is to be taken in Dirac and color space, whereas the flavor trace has been separated out.

We will calculate the induced QCD vacuum condensates from the saddle-point expansion, that is, we will work at the mean field level. This is consistent with the large N_c limit in the quark fields for a given model gluon two-point function. Under the mean field approximation, the dressed quark propagator $G(x-y)$ is substituted by $G_0(x-y)$ which has the decomposition

$$G_0^{-1}(p)=i\gamma\cdot p+\Sigma(p)=i\gamma\cdot pA(p^2)+B(p^2). \quad (12)$$

The $\Sigma(p)$ stands for the dressing self-energy of quarks and is defined as

$$\Sigma(p)=\Lambda^\theta B_0^\theta(p)=i\gamma\cdot p[A(p^2)-1]+B(p^2), \quad (13)$$

where the self-energy functions $A(p^2)$ and $B(p^2)$ are determined by the rainbow Dyson-Schwinger equations (DSEs)

$$[A(p^2)-1]p^2 = \frac{8}{3}\int\frac{d^4q}{(2\pi)^4}g^2D(p-q)\frac{A(q^2)p\cdot q}{q^2A^2(q^2)+B^2(q^2)}, \quad (14)$$

$$B(p^2) = \frac{16}{3}\int\frac{d^4q}{(2\pi)^4}g^2D(p-q)\frac{B(q^2)}{q^2A^2(q^2)+B^2(q^2)}. \quad (15)$$

Because the form of the gluon propagator $g^2D(s)$ in the infrared region is unknown, one often uses various model forms [15, 16, 17] as input parameters in the previous studies of the Rainbow DSE.

Within the DSE formalism, there are two qualitatively distinct solutions in Eq(14) and (15). The "Nambu-Goldstone" solution characterized by $B(p^2)\neq 0$ describes a phase : (1) chiral symmetry is dynamically broken for it provides a momentum dependent constituent quark mass $M(p^2)=B(p^2)/A(p^2)$; and (2) the dressed quarks are confined for the dressed quark propagator does not have a Lehmann representation[16]. The alternative "Wigner" solution characterized by $B(p^2)\equiv 0$ describes a phase with neither dynamical chiral symmetry breaking nor confinement. In this letter, we refer to the quark propagator in terms of the trivial solution

of the gap equation as the "perturbative" quark propagator $G_0^{per}(p^2)$ with only the vector part

$$[A'(p^2)-1]p^2 = \frac{8}{3}\int \frac{d^4q}{(2\pi)^4} g^2 D(p-q) \frac{p\cdot q}{q^2 A'(q^2))}. \tag{16}$$

This "perturbative" quark propagator can be seen as the expectation value of the operator $T[q_i(x)\overline{q}_j(y)]$ over the perturbative vacuum $|P\rangle$ at the mean field level in the framework of GCM.

From the GCM generating functional, it is now straightforward to calculate the vacuum expectation value(VEV) of any quark operator of the form

$$\mathcal{O}_n \equiv (\overline{q}_{j_1}\Lambda^{(1)}_{j_1 i_1} q_{i_1})(\overline{q}_{j_2}\Lambda^{(2)}_{j_2 i_2} q_{i_2})\cdots(\overline{q}_{j_n}\Lambda^{(n)}_{j_n i_n} q_{i_n}), \tag{17}$$

in the mean field vacuum. Here the $\Lambda^{(i)}$ stands for an operator in Dirac, flavor, and color space. Take the appropriate number of derivatives with respect to external source terms η_i and $\overline{\eta}_j$ of Eq. (9) and set $\eta_i = \overline{\eta}_j = 0$ [22], we can get

$$\langle \mathcal{O}_n \rangle = (-1)^n \sum_p (-)^p [\Lambda^{(1)}_{j_1 i_1} \cdots \Lambda^{(n)}_{j_n i_n} (G_0)_{i_1 j_{p(1)}} \cdots (G_0)_{i_n j_{p(n)}}], \tag{18}$$

where p stands for a permutation of the n indices. Once the dressing quark propagator $G_0(q^2)$ (We ignore the subscript 0 below)is determined, one can calculate the two quark condensate $\langle\overline{q}q\rangle$, the four quark condensate$\langle\overline{q}\Lambda^{(1)}q\overline{q}\Lambda^{(2)}q\rangle$, etc. in the mean field level. Since the functional integration over the gluon field A^a_μ is quadratic in the framework of GCM, one can perform the integration over gluon field analytically. Using the same shorthand notation for the typical Gaussian integrations as in Ref. [6], we have

$$\begin{aligned}
\int \mathcal{D}A e^{-\frac{1}{2}AD^{-1}A+jA} &= e^{\frac{1}{2}jDj} \\
\int \mathcal{D}A A e^{-\frac{1}{2}AD^{-1}A+jA} &= (jD)e^{\frac{1}{2}jDj} \\
\int \mathcal{D}A A^2 e^{-\frac{1}{2}AD^{-1}A+jA} &= [D+(jD)^2]e^{\frac{1}{2}jDj}
\end{aligned} \tag{19}$$

where D is the dressing gluon propagator and j^a_μ is the quark color current. Because the gluon vacuum average can be replaced by a quark color current $\overline{q}\gamma_\mu\frac{\lambda^a_C}{2}q$ together with the gluon two-point function D, one can perform the integration over the quark operators in the mean field vacuum as described above. In this way, one can in principle obtain the vacuum expectation of value for any gluonic fields. This technique provides an feasible way to evaluate the expectation value of the operators with low-dimensional gluon fields such as the mixed quark-gluon condensate $g\langle\overline{q}G_{\mu\nu}\sigma^{\mu\nu}q\rangle$ in GCM (Note that for the VEV of operators with high powers of gluonic fields, this procedure will get rather complex).

3. Mixed Tensor Susceptibility

With above preparation, the mixed tensor susceptibility can be calculated in the mean field level within this DSE model. The induced tensor susceptibilities χ, κ and ζ are defined through

$$\langle V|\overline{q}\sigma_{\mu\nu}q|V\rangle_Z = g_q\chi Z_{\mu\nu}\langle\overline{q}q\rangle \tag{20}$$

$$\langle V|\overline{q}g_c\frac{\lambda^a}{2}G^a_{\mu\nu}q|V\rangle_Z = g_q\kappa Z_{\mu\nu}\langle\overline{q}q\rangle \tag{21}$$

$$\langle V|\overline{q}g_c\gamma_5\widetilde{G}_{\mu\nu}q|V\rangle_Z = -ig_q\zeta Z_{\mu\nu}\langle\overline{q}q\rangle, \tag{22}$$

where $Z_{\mu\nu}$ stands for the external field, $\langle V|\cdots|V\rangle_Z$ denotes the VEV over the QCD vacuum at the presence of the external filed $Z_{\mu\nu}$ and $\widetilde{G}_{\mu\nu} = \frac{1}{2}\frac{\lambda^a}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta a}$. The nonzero VEVs of the operators above are due to the breakdown of Lorentz invariance in the presence of external constant field $Z_{\mu\nu}$. From the QCD partition function for quarks in Euclidean space in the presence of the external field , the formulae for evaluating these susceptibilities take the form

$$\chi\langle\overline{q}q\rangle = \frac{1}{6}\int d^4x\langle V|T[\overline{q}(x)\sigma_{\mu\nu}q(x),\overline{q}\sigma_{\mu\nu}q]|V\rangle = \frac{1}{6}\Pi_\chi(0) \tag{23}$$

$$\kappa\langle\overline{q}q\rangle = \frac{1}{6}\int d^4x\langle V|T[\overline{q}(x)g_c\frac{\lambda^a}{2}G^a_{\mu\nu}q(x),\overline{q}\sigma_{\mu\nu}q]|V\rangle = \frac{1}{6}\Pi_\kappa(0), \tag{24}$$

according to Ref. [6, 7]. Due to the fact that the vacuum susceptibilities reflect the nonperturbative structure of the QCD vacuum, $\Pi_\chi(0)$ and $\Pi_\kappa(0)$ on the right hand side of above Eqs.(23) and (24) should be subtracted by the corresponding perturbative contribution terms. Within the DSE formalism, the perturbative contribution to $\Pi_\chi(0)$ and $\Pi_\kappa(0)$ can be evaluated by the trivial quark propagator, namely the "perturbative" quark propagator in terms of the trivial Wigner solution to the dressed quark gap equations (14) and (15). This is a reasonable procedure because the Wigner solution of the dressed quark propagator describes a phase with neither DCSB nor confinement and the difference between the Nambu solution and the Wigner solution vanishes at short distance according to numerical studies. In fact, the Wigner solution can play the role of the perturbative dressed quark propagator has been used extensively in the study of thermal property of QCD within the DSE formalism[17], where the bag constant was defined as the difference of pressure between the true QCD vacuum and the perturbative QCD vacuum, which were evaluated by the Nambu-Goldstone solution and the Wigner solution to the quark propagator, respectively[18].

Therefore, we rewrite the Eqs. (23) and (24) as

$$
\begin{aligned}
\chi\langle\overline{q}q\rangle &= \frac{1}{6}\int d^4x\langle V|T[\overline{q}(x)\sigma_{\mu\nu}q(x),\overline{q}\sigma_{\mu\nu}q]|V\rangle^N \\
&\quad -\frac{1}{6}\int d^4x\langle P|T[\overline{q}(x)\sigma_{\mu\nu}q(x),\overline{q}\sigma_{\mu\nu}q]|P\rangle^W \\
&= \frac{1}{6}\Pi_\chi^{np}(0), \qquad (25)\\
\kappa\langle\overline{q}q\rangle &= \frac{1}{6}\int d^4x\langle V|T[\overline{q}(x)g_c\frac{\lambda^a}{2}G^a_{\mu\nu}q(x),\overline{q}\sigma_{\mu\nu}q]|V\rangle^N \\
&\quad -\frac{1}{6}\int d^4x\langle P|T[\overline{q}(x)g_c\frac{\lambda^a}{2}G^a_{\mu\nu}q(x),\overline{q}\sigma_{\mu\nu}q]|P\rangle^W \\
&= \frac{1}{6}\Pi_\kappa^{np}(0). \qquad (26)
\end{aligned}
$$

By substituting the "perturbative" quark propagator $G^{per}(p^2)$ to Eq.(18), the determination of the expectation value of the T-product operators in terms of quark fields over the perturbative vacuum state $|P\rangle$ can be performed self-consistently within the GCM formalism. It should be noted that the evaluation of $\chi\langle\overline{q}q\rangle$ in Ref.[11] is consistent with Eq.(25) because in this special case the subtraction terms to Eq. (25)has zero contribution to $\chi\langle\overline{q}q\rangle$ due to $B'(p^2)\equiv 0$.

Using Eq. (19), the expression for VEV of above T-product operator including gluonic fields is converted to the VEV for the product with the form of (17) in terms of six quark fields and eight quark fields. According to Eq. (18), we have

$$
\begin{aligned}
&\frac{1}{6}\int d^4x\langle V|T[\overline{q}(x)g_c\frac{\lambda^a}{2}G^a_{\mu\nu}q(x),\overline{q}\sigma_{\mu\nu}q]|V\rangle^N = \\
&\quad -\frac{4}{3}i\int dx^4\int dz^4 g^2\left[\partial^x_\mu D(z-x)\right]\times \\
&\quad \Big\{tr_D\left[G(x-z)\gamma_v G(z-0)\sigma_{\mu\nu}G(0-x)\right] \\
&\quad +tr_D\left[G(x-0)\sigma_{\mu\nu}G(0-z)\gamma_v G(z-x)\right]\Big\} \\
&\quad -2i\int dx^4\int dz_1^4\int dz_2^4 g^2 D(z_1-x)g^2D(z_2-x)\times \\
&\quad \Big\{tr_D\left[G(x-z_1)\gamma_\mu G(z_1-z_2)\gamma_\nu G(z_2-0)\sigma_{\mu\nu}G(0-x)\right] \\
&\quad +tr_D\left[G(x-z_1)\gamma_\mu G(z_1-0)\sigma_{\mu\nu}G(0-z_2)\gamma_\nu G(z_2-x)\right] \\
&\quad +tr_D\left[G(x-0)\sigma_{\mu\nu}G(0-z_1)\gamma_\mu G(z_1-z_2)\gamma_\nu G(z_2-x)\right]\Big\}. \qquad (27)
\end{aligned}
$$

Substituting $G^{per}(x-y)$ for $G(x-y)$, the similar expression for the VEV of same operator over the perturbative vacuum can be obtained. After Fourier transformation, we find the first

part of right hand side of (27) is zero and the final result for the mixed tensor susceptibility in the momentum space takes the form

$$\begin{aligned}
\kappa\langle\overline{q}q\rangle &= \frac{1}{16\pi^2}\int dss[\frac{B(s)}{Z(s)}]^2[\frac{27}{8}B^2(s)+\frac{27}{2}sA(s)(2-A(s))] \\
&-\frac{9}{32\pi^5}\int dsdt\int_{-1}^{1}dxst\sqrt{1-x^2}g^2D(s,t,\sqrt{st}x)Z^{-2}(s)Z^{-1}(t) \\
&\times B(s)\Big\{B(s)B^2(t)+B(t)A(s)A(t)\sqrt{st}x-\big[A(s)-1\big] \\
&\times\big[2A(s)B(t)\sqrt{st}x-A(t)B(s)\big]\Big\} \\
&+\frac{3}{32\pi^5}\int dsdt\int_{-1}^{1}dxst\sqrt{1-x^2}g^2D(s,t,\sqrt{st}x)Z'^{-2}(s)Z'^{-1}(t) \\
&\times A'^2(s)A'(t)\big[A'(t)-1\big]\big[st-4stx^2\big] \\
&-\frac{3}{32\pi^5}\int dsdt\int_{-1}^{1}dxst\sqrt{1-x^2}g^2D(s,t,\sqrt{st}x)Z^{-2}(s)Z^{-1}(t) \\
&\times A^2(s)A(t)\big[A(t)-1\big]\big[st-4stx^2\big], \qquad (28)
\end{aligned}$$

where $Z(s)=sA^2(s)+B(s)$ and $Z'(s)=sA'^2(s)$.

It should be noted that to get this expression the Dyson-Schwinger equation (11) has been used again. The UV divergence of Eq. (27) can be illustrated by a simple analytical confining model $g^2D(p-q)=\frac{3}{16}(2\pi)^4\eta^2\delta^4(p-q)$[19]. In this model, the expression for (27) takes a relative simple form

$$\begin{aligned}
&\frac{1}{6}\int d^4x\langle V|T[\overline{q}(x)g_c\frac{\lambda^a}{2}G^a_{\mu\nu}q(x),\overline{q}\sigma_{\mu\nu}q]|V\rangle^N = \\
&\quad\frac{1}{16\pi^2}\int dss[\frac{B(s)}{Z(s)}]^2\big[\frac{27}{8}B^2(s)+\frac{27}{2}sA(s)(2-A(s))\big] \\
&\quad-\frac{36\eta^2}{16\pi^2}\int dssZ^{-3}(s)\big[B^4(s)+sB^2(s)A(s)\big] \\
&\quad+\frac{36\eta^2}{16\pi^2}\int dss^3Z^{-3}(s)\big[A(s)-1\big]A^3(s). \qquad (29)
\end{aligned}$$

The Nambu-Goldstone solution for this model is

$$A(p^2) = \begin{cases} 2, & p^2<\frac{\eta^2}{4}, \\ \frac{1}{2}(1+\sqrt{1+\frac{2\eta^2}{p^2}}), & \text{otherwise}, \end{cases} \qquad (30)$$

$$B(p^2) = \begin{cases} \sqrt{\eta^2-4p^2}, & p^2<\frac{\eta^2}{4}, \\ 0, & \text{otherwise}. \end{cases} \qquad (31)$$

The alternative Wigner solution takes the form

$$B'(p^2) \equiv 0, \quad A'(p^2) = \frac{1}{2}(1 + \sqrt{1 + \frac{2\eta^2}{p^2}}). \tag{32}$$

Due to $A(s) - 1 \sim c/s$ for $s \to \infty$ according to (30), the last term of right hand side of Eq. (29) is logarithmic divergent. In addition, replacing $A(s)$ with $A'(s)$ to (29), there still exits logarithmic divergence due to $A'(s)$ having the same behavior as $A(s)$ in the large energy region. Because the vector part $A(s)$ as well as the scalar part $B(s)$ both reflect the nonperturbative information in the low energy region, it is more reasonable to subtract the corresponding perturbative part rather than to simply ignore this divergent term in (29).

With the effective subtraction of the perturbative contribution, there is no UV divergence in above integrations. Actually, the subtraction procedure guarantee $\kappa\langle\overline{q}q\rangle$ the role of the order parameter for QCD chiral phase transition because it becomes zero when QCD undergoes a phase transition from the Nambu-Goldstone phase to Wigner phase (That means $A(p^2) \to A'(p^2)$ and $B(p^2) \to 0$).

4. Results and Discussions

The determination of the mixed tensor susceptibility is based on the same effective gluon propagator models $g^2D(s)$ which had been used in Ref. [12, 20]. In general, the quark-quark interaction $g^2D(s)$ has the form

$$g^2D(s) = \frac{4\pi\alpha(s)}{s}, \tag{33}$$

where $s = p^2$. Two popular quark-quark interaction models with two parameters for $\alpha(s)$ are investigated here:

$$\alpha_1(s) = 3\pi s\chi^2 \frac{e^{-\frac{s}{\Delta}}}{4\Delta^2} \tag{34}$$

$$\alpha_2(s) = d\pi s \frac{\chi^2}{s^2 + \Delta}, \tag{35}$$

where $d = \frac{27}{12}$. The two low-momentum parameters, the stength parameter χ and the range parapmeter Δ , are varied with the pion decay constant fixed at 87 MeV which is more appropriate in the chiral limit rather than the pion's mass-shell value of 93 MeV. Noted that the above quark-quark interactions dominate for small s and simulate the infrared enhancement and confinement. Because the effective quark-quark interactions(34,35) have a finite range in momentum space, the momentum integral for the calculation of the quark condensate

$$\langle\overline{q}q\rangle = -\frac{3}{4\pi^2}\int_0^\infty dss\frac{B(s)}{sA^2(s) + B^2(s)}, \tag{36}$$

is finite. According to [12], the obtained values of the chiral low energy coefficients L_i following Ref.[20] based on both ansatz (34) and (35) are compatible with the phenomenological values.

The model ansatz (35) has been successfully used to investigate the space structure of the non-local quark condensate $\langle\overline{q}(x)q(0)\rangle$ in Ref.[21] within GCM formalism. It should be stressed in this context that our interactions are not renormalizable due to using the bare quark-gluon vertex within the rainbow DSE formalism. Therefore, the scale at which a condensate is defined in our calculation is a typical hadronic scale, which is implicitly determined by the model quark-quark interaction and the solutions of the DSEs for the dressed quark propagator. The similar case is the determination of the vacuum condensate in the instanton liquid model[23] where the scale is set by the inverse instanton size.

To check the sensitivity of the mixed tensor susceptibility on the forms of quark-quark interactions, the above models with different sets of parameters χ and Δ are investigated below, where the results for the quark condensate are also given.

Table 1. The value of $\kappa\langle\overline{q}q\rangle$ for model 1 with two sets of different parameters.

$\Delta(\mathrm{GeV}^2)$	$\chi(\mathrm{GeV})$	$-\langle\overline{q}q\rangle^{\frac{1}{3}}(\mathrm{MeV})$	$\kappa\langle\overline{q}q\rangle(\mathrm{GeV}^4)$	$\kappa(\mathrm{GeV})$
0.2	1.55	213	$2.5 * 10^{-3}$	-0.26
0.02	1.39	170	$2.9 * 10^{-3}$	-0.59
0.002	1.30	149	$1.4 * 10^{-3}$	-0.41

Table 2. The value of $\kappa\langle\overline{q}q\rangle$ for model 2 with four sets of different parameters.

$\Delta(\mathrm{GeV}^4)$	$\chi(\mathrm{GeV})$	$-\langle\overline{q}q\rangle^{\frac{1}{3}}(\mathrm{MeV})$	$\kappa\langle\overline{q}q\rangle(\mathrm{GeV}^4)$	$\kappa(\mathrm{GeV})$
$1 * 10^{-1}$	1.77	290	$4.7 * 10^{-3}$	-0.19
$1 * 10^{-2}$	1.33	250	$3.6 * 10^{-3}$	-0.23
$1 * 10^{-4}$	0.95	217	$3.0 * 10^{-3}$	-0.29
$1 * 10^{-6}$	0.77	204	$3.1 * 10^{-3}$	-0.36

In Table 1 we display the values for $\langle\overline{q}q\rangle$ and $\kappa\langle\overline{q}q\rangle$ based on model 1 with three sets of parameters and in Table 2 the same quantities with four sets of parameters based on model 2. In both cases, the obtained values for $\langle\overline{q}q\rangle$ are compatible with the standard phenomenological value in SVZ sum rules, whereas the mixed tensor susceptibility $\kappa\langle\overline{q}q\rangle$ is much small. The previous estimation of $\kappa\langle\overline{q}q\rangle$ obtained In Ref. [6] is 0.10 GeV^4. Actually, the value of the quark condensate tensor susceptibility $\chi\langle\overline{q}q\rangle$ obtained within GCM formalism [11] is also very small compared with the estimation based on SVZ sum rules . In fact, different versions of SVZ sum rules have given very different values for $\chi\langle\overline{q}q\rangle$ in previous studies [6, 8, 9]. Therefore, it shows that the induced vacuum condensates have very little impact on the determination of

the nucleon tensor charge from the theoretical formalism of DSEs.

In summary, we have investigated the mixed tensor susceptibility at the mean field level in the framework of GCM/DSE formalism. In the calculations, the vacuum matrix elements for the operator in terms of quark and gluonic fields can be obtained by substituting the gluonic fields with the quark color current operator and the model gluon propagator which describes the effective quark-quark interaction within the GCM formalism. To subtract the perturbative contribution to the expression for the mixed tensor susceptibility, the Wigner solution to the quark gap equation was used self-consistently in this formalism. Using different quark-quark interaction models, we find that the mixed tensor susceptibility as well as the quark condensate tensor are both very small susceptibility does within DSE formalism. Therefore, we get the conclusion that the induced vacuum condensates have little effect on the determination of the nucleon tensor charge. Finally, we want to tress that above approach can also be used to investigate the other mixed susceptibility of the QCD vacuum.

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